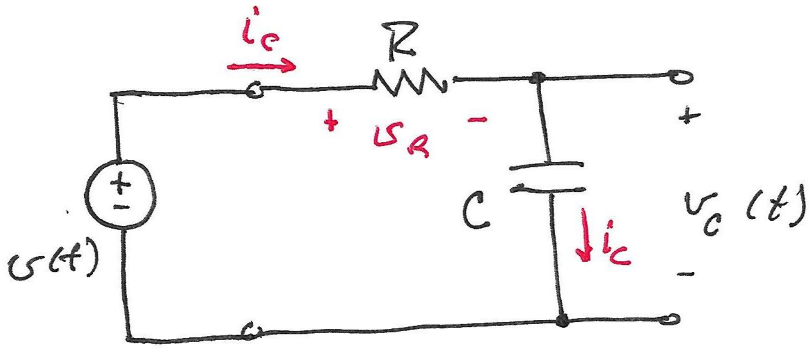


$$i_c = C\dot{v}_c$$

$$v_R = R\,i_c = RC\dot{v}_c$$

KVL: $v_R + v_c = v$

$$RC\dot{v}_c + v_c = v$$

$$\dot{v}_c + \frac{1}{RC}v_c = \frac{V_s}{RC}$$

$$v_c(t) = K_1 e^{\alpha t} + K_2$$

$$\frac{1}{RC}K_2 = \frac{V_s}{RC}$$

$$K_2 = V_s$$

$$v_c(t) = K_1 e^{\alpha t} + V_s$$

$$\alpha K_1 e^{\alpha t} + 0 + \frac{1}{RC} K_1 e^{\alpha t} + \cancel{\frac{1}{RC} V_s} = \cancel{\frac{V_s}{RC}}$$

$$\alpha \cancel{K_1} \cancel{e^{\alpha t}} + \frac{1}{RC} \cancel{K_1} \cancel{e^{\alpha t}} = 0$$

$$\alpha + \frac{1}{RC} = 0$$

$$\Rightarrow \quad \alpha = -\frac{1}{RC}$$

$$v_c(t) = K_1 e^{-\frac{1}{RC} t} + V_s$$

$$v_c(t_0) = 0$$

Let $t_0 = 0$

$$v_c(0) = 0$$

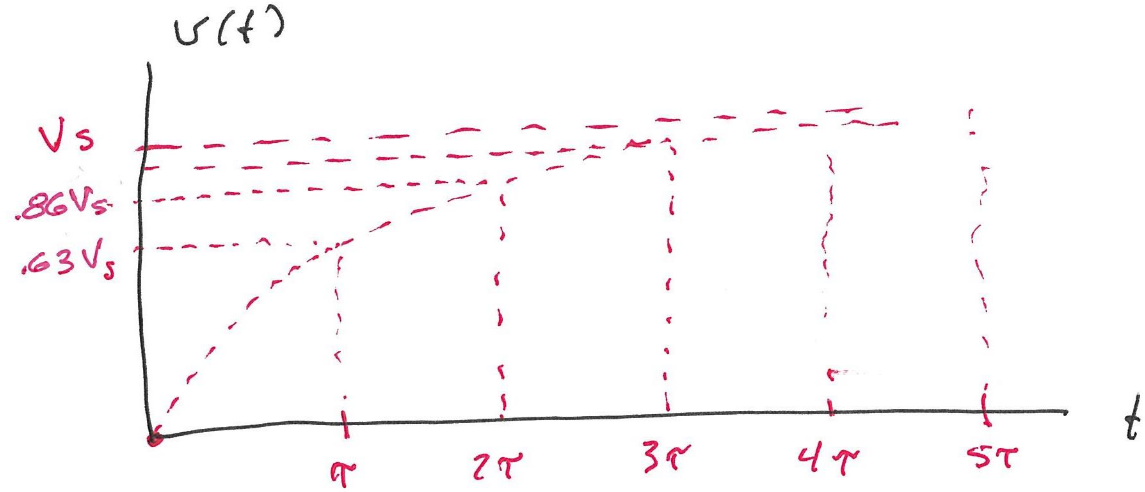
$$v_c(0) = K_1 \underbrace{\cancel{e^0}}_{1} + V_s$$

$$= K_1 + V_s = 0 \quad \Rightarrow \quad K_1 = -V_s$$

$$\boxed{v_c(t) = -V_s e^{-\frac{1}{RC} t} + V_s \quad \text{for } t \geq 0}$$

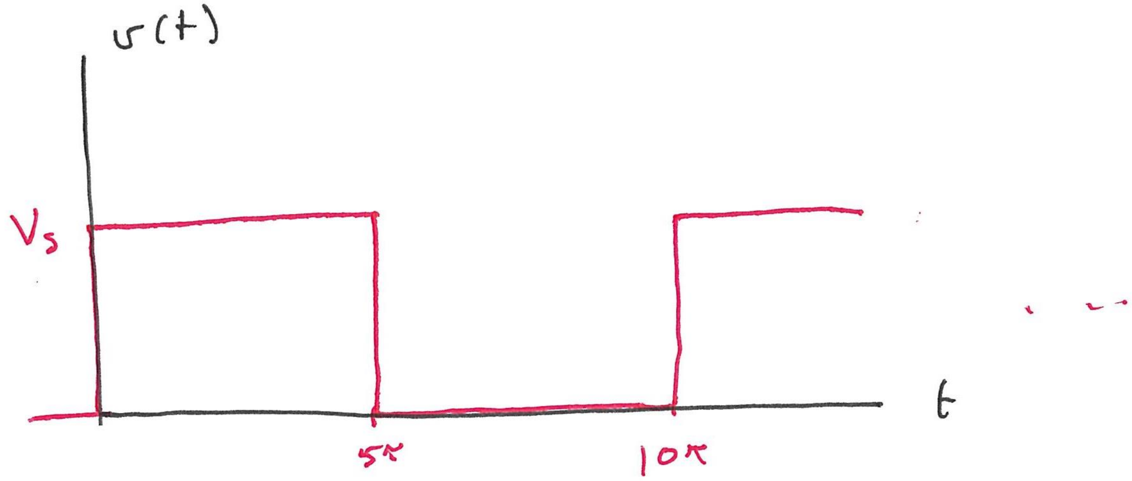
The time at which $e^{-\frac{1}{RC}t} = e^{-1}$ is called the time constant

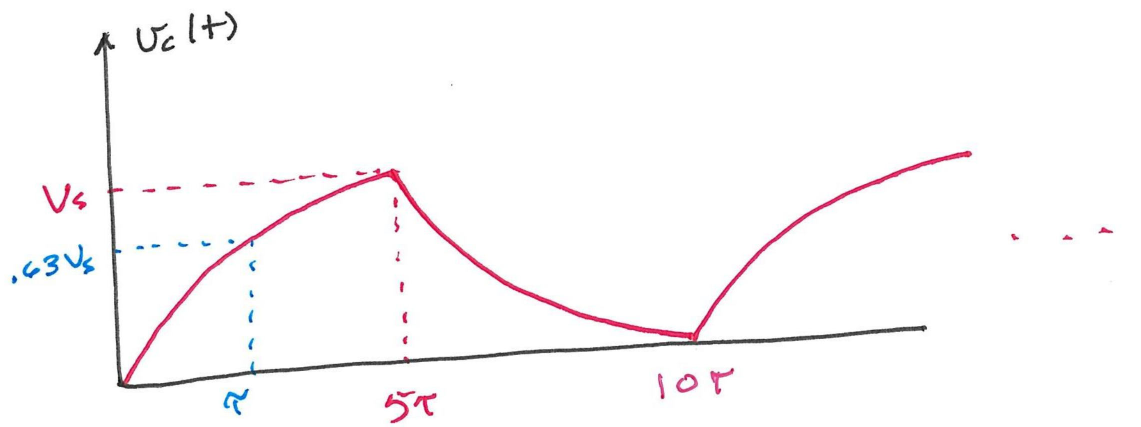
$$\tau = RC$$

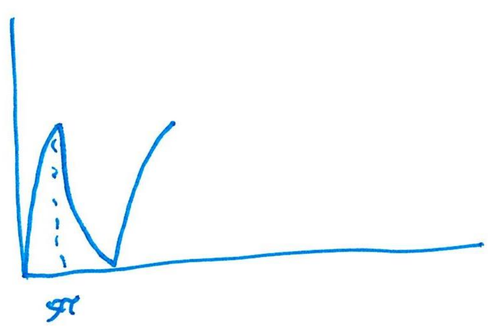


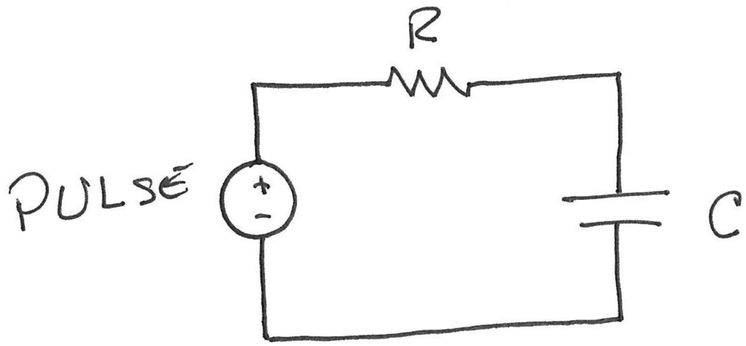
Rule of Thumb:

The signal has reached its final value when $t = 5\tau$.


v(t)
Vs
t
5τ
10τ


vc(t)
Vs
.63Vs
τ
5τ
10τ

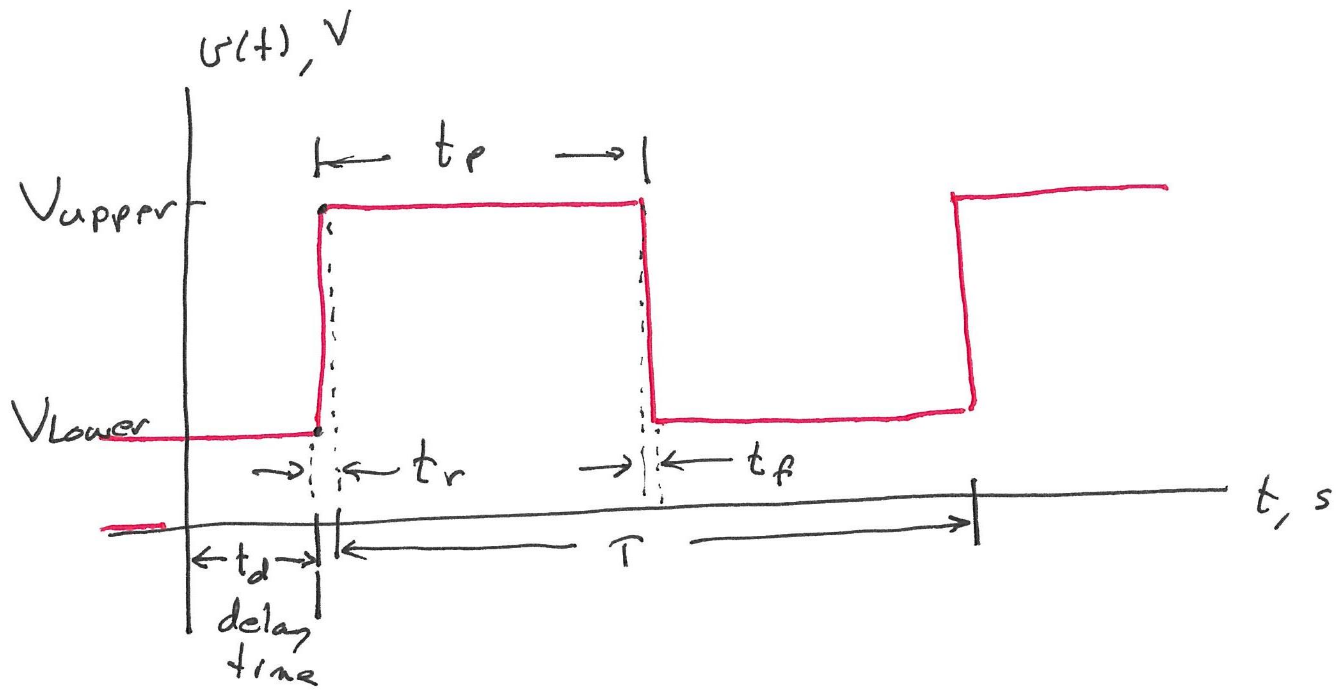

5τ



PULSE(   )

.TRAN 50m

Pulse Waveform



V_{lower}

V_{upper}

t_d delay time

t_r rise time

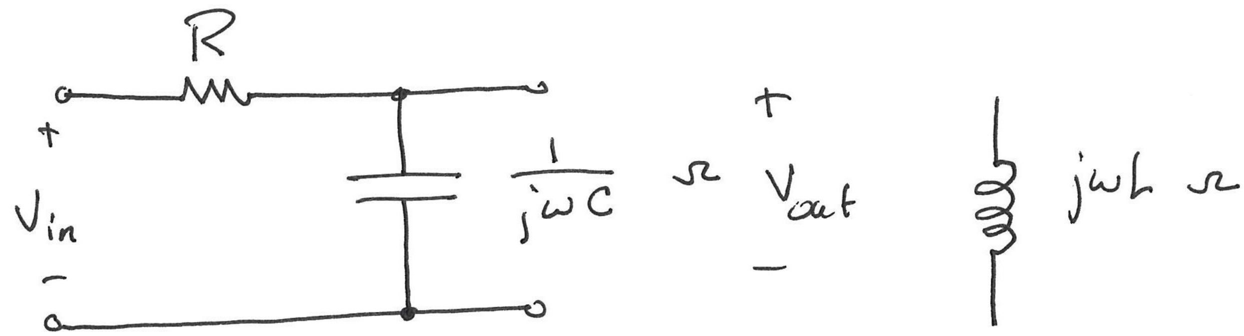
t_f fall time

t_p pulse width

T period

For an RC circuit, $\tau = RC$ seconds

For an RL circuit, $\tau = L/R$ seconds



$$-j\frac{1}{\omega C}$$

$$V_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{in} \cdot \frac{j\omega C}{j\omega C}$$

$$= \frac{1}{j\omega RC + 1} V_{in}$$

$\frac{V_{out}}{V_{in}} = \frac{1}{j\omega RC + 1}$ is called the transfer function

$$H(\omega) \triangleq \frac{V_{out}}{V_{in}}$$

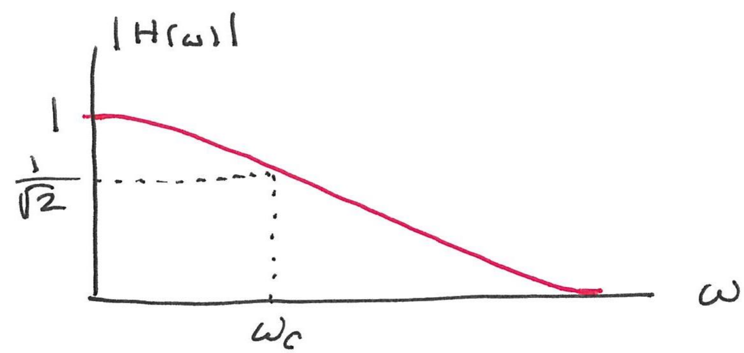
phasor rep. for V_{out}

phasor rep. for V_{in}

$$|H(\omega)| = \frac{|1|}{|j\omega RC + 1|} = \frac{1}{\sqrt{(1)^2 + (\omega RC)^2}}$$

$$\lim_{\omega \to 0} |H(\omega)| = \frac{1}{\sqrt{1^2}} = 1$$

$$\lim_{\omega \to \infty} |H(\omega)| = \frac{1}{\sqrt{(\omega RC)^2}} = \frac{1}{\omega RC} = 0$$

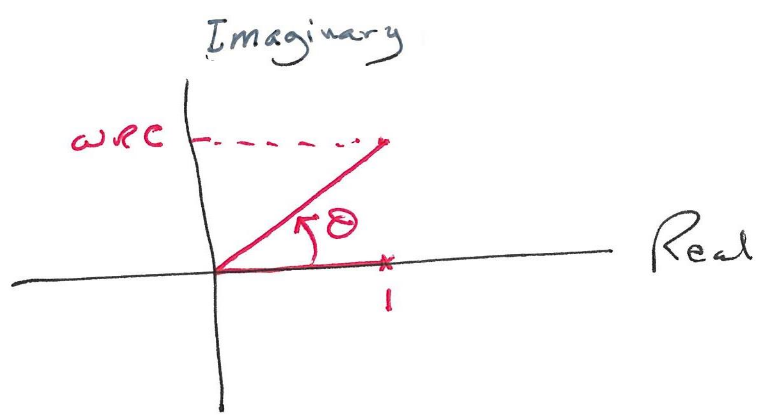


Low Pass filter

Phase angle of $H(\omega)$

$$\angle H(\omega) = \angle 1 - \angle j\omega RC+1$$

$$\angle 1 = 0^\circ$$



$$\angle j\omega RC+1 = \tan^{-1} \frac{\omega RC}{1} = \tan^{-1} \omega RC$$

$$\angle H(\omega) = -\tan^{-1} \omega RC$$

$$\lim_{\omega \to 0} \angle H(\omega) = -\tan^{-1} 0 = 0$$

$$\lim_{\omega \to \infty} \angle H(\omega) = -\tan^{-1} \infty = -90^\circ$$